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HEAT TRANSFER AND EQUILIBRIUM TEMPERATURE ON A THIN PLATE IN HYPERSONIC FLOW WITH STRONG INTERACTION

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Аннотация—В статье сообщается о методике и результатах экспериментальных исследований локальных коэффициентов теплоотдачи и равновесных температур на тонкой плоской пластине, обтекаемой воздухом при нулевом угле атаки, больших числах Маха и низких числах Рейнольдса.

NOMENCLATURE

x,	distance from a leading edge downwards flow;
t,	plate thickness;
Te_x,	local equilibrium temperature on a plate;
T_{00},	stagnation temperature of incoming flow;
T,	static temperature of incoming flow;
T_w,	plate temperature;
u,	velocity of incoming flow;
$Re_x = \frac{u\rho x}{\mu}$,	Reynolds number composed according to incoming-flow parameters and co-ordinate x;
M,	Mach number of incoming flow;
$\chi = M^3\sqrt{C}/\sqrt{Re_x}$,	interaction parameter;
$\bar{T}_w = T_w/T_{00}$,	temperature factor;
$\tau_x = \frac{Te_x - T}{T_{00} - T}$,	local temperature-recovery factor;
$St_x = \frac{\alpha_x}{\rho c_p u}$,	local Stanton number composed according to the parameters of incoming flow and local heat-transfer coefficient, α_x;
$Kn_t = \lambda/t$,	Knudsen number which is the ratio of the mean free path of incoming flow to a plate thickness;
$c = \frac{\mu_w}{\mu} \cdot \frac{T}{T_w}$;	
μ_w,	air viscosity at T_w.

INTRODUCTION

IT IS KNOWN THAT in hypersonic flow past a thin plate (when the plate thickness is much less than the mean free path of molecules in the incoming flow), the interaction of a shock wave, formed near the leading edge of a plate, with the boundary layer [1] leads to an increase of the induced pressure, heat transfer and friction. Depending on the value of the interaction parameter, we can distinguish along the stream different regions, viz. of strong, mean and weak interaction, and of a gradient-free boundary layer [1, 2] (See Fig. 1).

The shock wave and the boundary layer, where the flow may be described by the Navier–Stokes–Fourier continuum equations, are formed at a distance from the leading edge of the plate $x \approx N\lambda M$, where $N \approx 10$ [2, 3]. In the region $0 \leq x \leq N\lambda M$, the flow is characterized by slip and should be analysed in terms of the molecular kinetic gas theory. Some experimental works on heat transfer on a plate in hypersonic flow have established that the influence of slip upon the distribution of heat fluxes vanishes at a distance x from the leading edge of the plate, that corresponds to the parameter

$$M\sqrt{(C)}/\sqrt{(Re_x)} \approx 0{\cdot}3 \text{ [3].}$$

The survey of published works devoted to the experimental and theoretical investigations of heat-transfer processes in the region of a strong interaction where the gas possesses the properties of the Navier–Stokes–Fourier continuum [1, 2, 4] shows that, in general, the study of these

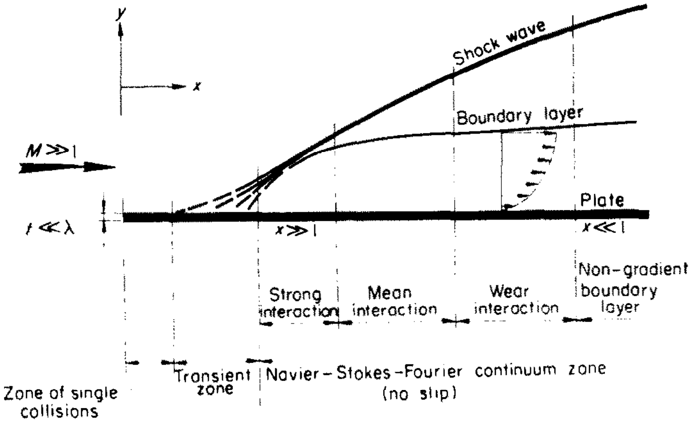


FIG. 1. Gas flow pattern near a plate.

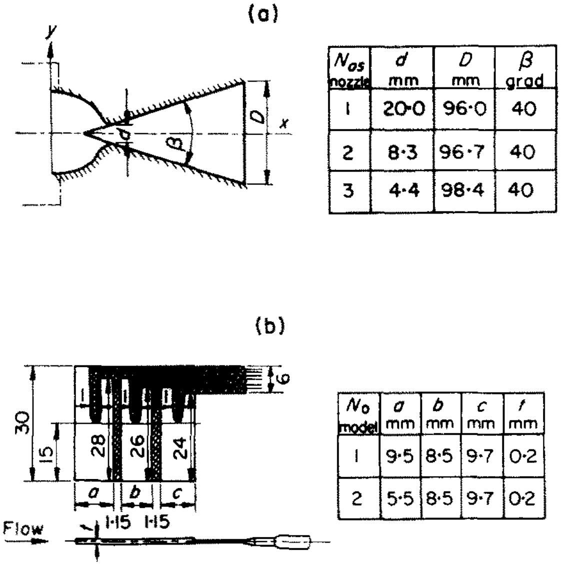
processes may be considered as completed. However, the subject of the equilibrium temperature in the region of a strong interaction remains, as yet, unexplored. Also, the experimental investigations of the distribution of the heat fluxes have so far been carried out only at small values of the temperature factor, viz. $T_w = 0{\cdot}1$–$0{\cdot}2$. It is, therefore, of interest to check experimentally the conclusions of the theory of the strong interaction with regard to the distribution of heat fluxes at large values of the temperature factor.

The present work is devoted to the experimental investigation of heat transfer and equilibrium temperature on a flat plate in the region of strong interaction ($\chi \gg 1$) when $M\sqrt{C}/\sqrt{Re_x} \lesssim 0{\cdot}3$, i.e. when there is no slip. The experiments have been conducted at $M = 3{\cdot}5$–$8{\cdot}1$ for the incoming flow. The Knudsen numbers computed as the ratio of the mean free path of the molecules in the incoming flow to the plate thickness were greater than unity in all the experiments. In the experiments on heat transfer the values of the temperature factor were $T_w = 0{\cdot}62$–$0{\cdot}83$ (which corresponds to the stagnation temperatures of $T_{00} = 550°$–$400°$K).

METHODS OF INVESTIGATION

A number of investigations have been carried out in a vacuum wind tunnel, and described in references 5 and 6. Dried air served as the working substance. The flows were produced with the aid of three conic nozzles, whose construction is shown in Fig. 2(a).

As the working stream the isoentropic core of the flow in the nozzle outlet was used, where the longitudinal gradient of the velocity was quite negligible due to the quickly increasing thick boundary layer on the nozzle walls. In the nozzle outlet the mean relative change in the Mach number along the flow over a distance of 1 cm did not exceed 1–2 per cent in all the



N_{os} nozzle	d mm	D mm	β grad
1	20·0	96·0	40
2	8·3	96·7	40
3	4·4	98·4	40

No model	a mm	b mm	c mm	f mm
1	9·5	8·5	9·7	0·2
2	5·5	8·5	9·7	0·2

FIG. 2. (a) Design and dimensions of conic nozzles;
(b) Design and dimensions of a sectional plate.

experiments. In all the experiments the diameter of the working flow was approximately 30–40 mm, the static pressure was within

$$p = 2\times 10^{-2}\text{–}2\times 10^{-3}\ \text{mm Hg}.$$

In each experiment the dimensions and parameters of the working flow were determined by measuring the velocity and temperature fields in the gas moving along the nozzle [5].

In the supersonic flow the velocity fields were measured by the Pitot tubes which were first calibrated in order to take into consideration the effect of rarefaction upon their indications. The calibration results agree satisfactorily with the data of similar investigations [7]. The temperature fields in the supersonic flow were measured by thermocouple probes, whose descriptions were given in [6]. In each experiment the values of the static temperature T and static pressure p in the working flow were calculated using the Mach number measured by the Pitot tube and by the stagnation parameters p_{00}, T_{00} measured by the corresponding probes installed in front of the nozzle. The pressure and temperature gauges were mounted in a three-directional traverse gear, whose degree of accuracy for recording linear dimensions was $\pm 0{\cdot}1$–$0{\cdot}05$ mm. The accuracy of the adjustment of the angle of incidence was checked by the angle meter (and it was equal to ± 2 min). The pressures were registered by vacuum meters, type ВИТ–1, ВТ–2 and U-shaped liquid manometers which were filled with vacuum oil.

The experimental investigations into the kinetics of phase conversions of dry air in supersonic nozzles [8, 9] have shown that condensation or freezing occur at the states which correspond to a considerable supersaturation and take place not stepwise but gradually. According to the experiments in [9], the phase conversions of dry air begin when the supercooling approaches the value of 10–20 degC. The analysis has shown that in all the working streams used, even with $M = 7{\cdot}5$–$8{\cdot}1$, the phase conversions (condensation or freezing processes) were absent, since the subcooling of the air did not exceed 5–10 degC (see Fig. 3). The processes of the adiabatic air expansion in conic nozzles No. 1, No. 2 and No. 3 under the main (more typical) operating conditions are depicted in Fig. 3 in the co-ordinate system p–T. The values of the Mach numbers are given for the states corresponding to the end of the air expansion in the nozzle.

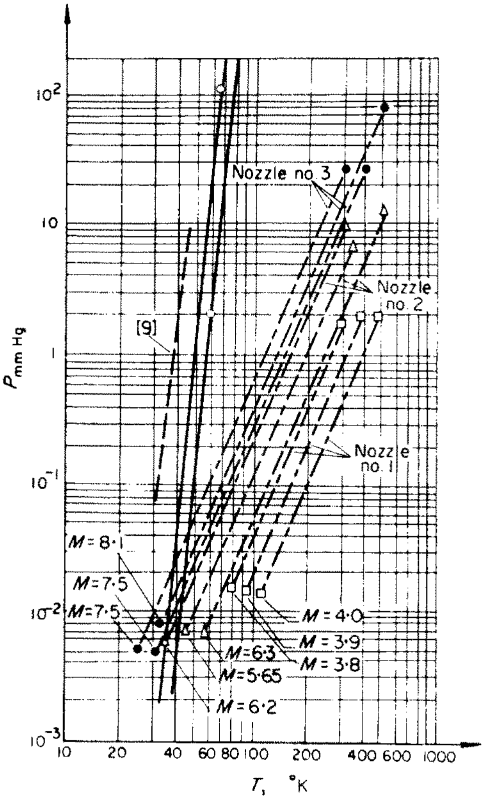


FIG. 3. Phase conversions in working flows;
—·—·— processes of isoentropic expansion of air in nozzles;
——— lines of saturation of nitrogen and oxygen in [10];
○ triple points;
– – – – boundary of the phase conversions start in supersonic nozzles from experiments in [9].

In order to study the local heat-transfer coefficients and equilibrium temperatures, two sectional plates were used. They were made of chemically pure silver, and their construction and sizes are given in Fig. 2(b). The sizes of the plates were chosen to fit the dimensions of the working flow, i.e. the plate itself and a boundary layer growing around it would be within the isoentropic core of the flow. In the middle of each section of the plate, there was, welded in, a chromel–coppel thermocouple, made of wires 0·1–0·15 mm in diameter, and which were lead out through a groove 0·5–0·9 mm wide. The groove was filled with a quick hardening solution of resin эд-5 in polyethylenepoliamine. Outside the plates, the wires of the thermocouples were also covered with this solution. The body of the solidified solution, with the wires inside it, formed a strong holder. The sections of the plates were stuck together with the same solution. After the plates had been gummed together and the thermocouples mounted, the plates were ground and polished. The solidified solution of the resin has good thermal properties ($K = 0{\cdot}07$–$0{\cdot}10$ kcal/m h degC) so that under the conditions of the experiments the heat flow from one section to another could be neglected.

The heat-transfer coefficient of the sections of the plates was defined by the transient method, whereby the change in the heat content of the body at a uniform temperature is taken as equal to the total heat flow into the body. Under the conditions of our experiments it was possible to assume that each section of the plate had uniform temperature distribution, since

$$Bi = \frac{a \cdot l}{k} < 10^{-4},$$

and therefore:

$$c\gamma V \frac{\mathrm{d}T_w}{\mathrm{d}\tau} = a \cdot S(T_e - T_w) \pm Q_{\mathrm{rad}} \qquad (1)$$

where c is the specific heat and γ the specific weight of silver; V is the volume of a section; S the total heat-transfer surface of the section; a the heat-transfer coefficient of the section; T_e the equilibrium temperature of the section; T_w the section temperature at a given time; Q_{rad} the radiant heat flow; τ the time; and k the heat conductivity of silver. In all the experiments the temperature of the cooled nozzle walls and of the plate sections differed only slightly from each other. Therefore, the radiant heat flow was negligibly small as compared with the heat flow due to convection. The estimates showed that

$$\frac{Q_{\mathrm{rad}}}{a \cdot S \cdot (T_e - T_w)} < 0{\cdot}01,$$

and therefore equation (1) may be written:

$$a = \tfrac{1}{2}\, c\gamma t \frac{\mathrm{d}\ln(T_e - T_w)}{\mathrm{d}\tau} \qquad (2)$$

where t is the thickness of the section of the plate. During the measurements of the heat-transfer coefficient, the sectional plate, initially cold, was quickly introduced into the gas stream in which it was gradually warming up. The changes in temperature of each section were read by a high-sensitivity recording oscillograph. The dependence of the logarithm of the excess temperature on time in the formula

$$\ln(T_e - T_w) = f(\tau)$$

was obtained on the basis of the measured dependence of temperature upon time for each section of the plate. Heat-transfer coefficients were determined by means of these charts and equation (2). The duration of each experiment was such that the change in the temperature factor T_w in time did not exceed 2–6 per cent.

The experiments on the heat-transfer coefficients were preceded by those on the equilibrium temperature of the plate sections. The steady-state method was used to determine the equilibrium temperature. The plate was introduced into the flow and left there until the temperature of each section ceased to change with time. In order to secure high accuracy, the experiments on the equilibrium temperatures were conducted without heating air in the electric heater. The stagnation temperature in the working stream was thus equal to the room temperature. The effects of radiative heat transfer could be neglected, since in these experiments the temperatures of the walls of the nozzle, the barocamera and the plate sections differed only slightly from each other.

In our experiments on the local heat-transfer coefficients and temperature-recovery coefficients, mean values of these coefficients were, generally speaking, measured for each section. On the basis of the analysis of the distribution law of the heat-transfer coefficients along the plate in the region of a strong interaction [2], it was accepted that for the second and third sections of the plate the values of these coefficients may be considered to be local for the middle point of each section. The consideration of the conditions, under which the second and third sections of the plate worked in all the experiments, showed that the use of the theory of strong interaction in that case was fully justified.

We did not use the experimental data obtained for the first section for our analysis of the distribution of heat fluxes and equilibrium temperatures in the region of strong interaction. Over a considerable part of the first section, slip occurred ($M\sqrt{C}/\sqrt{Re_x} > 0{\cdot}3$) in all the tests in agreement with the theoretical estimates. These estimates were well confirmed when the results for heat transfer obtained for the first section were compared with the conclusions of the theory of strong interaction.

Special experiments carried out on the plates of various dimensions showed that the results of the present investigations may be applied to the case of a flat flow (i.e. of an infinite flat plate).

Estimates of the errors showed that when defining the local heat-transfer coefficients the maximum errors could be ±10–15 per cent; when determining the local temperature-recovery factors ±3 per cent; the Stanton numbers ±15–20 per cent; the Mach numbers ±1–2 per cent.

Special experiments were conducted to define possible errors in measuring the coefficients of both the heat transfer and temperature recovery, due to the inaccuracy in the adjustment of the angle of incidence. In these experiments the quantities under consideration were measured while the incidence of the plate was changed in the range ±5°, the other conditions remaining the same. The experimental results showed that the accuracy in the adjustment of the plate was quite sufficient to relate the present results to the case of a zero-incidence angle.

RESULTS

The data of the experimental investigations are given in Fig. 4. The results of the experimental measurements of the temperature recovery factors are given in the co-ordinate system x–r_x in Chart I (Fig. 4). They were obtained for the second and third sections, and are within the range of the parameter $M\sqrt{C}/\sqrt{Re_x} \approx 0{\cdot}3$–$0{\cdot}16$.* For the whole range of Mach numbers of the incoming flow ($M = 3{\cdot}5$–$7{\cdot}8$) and the interaction parameters ($\chi = 3$–18) the local recovery factors were approximately equal to $r_x = 0{\cdot}95$. In all the experiments the values of the recovery factors for the first section ($M\sqrt{C}/\sqrt{Re_x} > 0{\cdot}3$) were noticeably higher than $r_x = 0{\cdot}95$, which fact is explained by the effect of slip on the equilibrium temperture. These data are not shown in Chart I (Fig. 4).

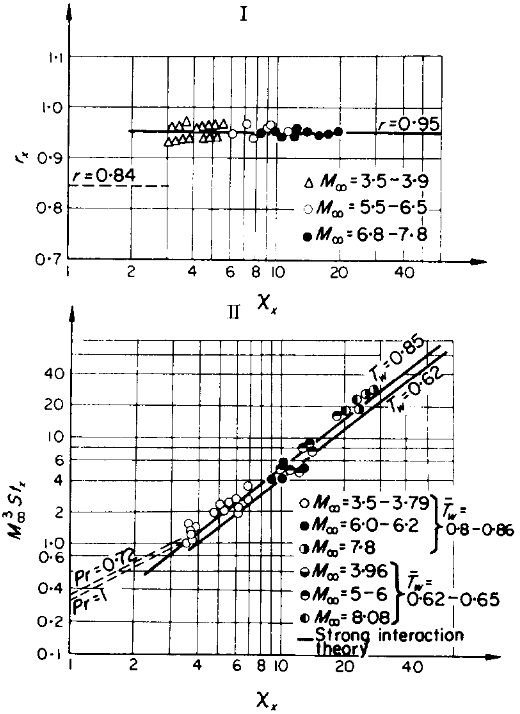


FIG. 4. Recovery factors (Chart I) and heat-transfer coefficients (Chart II) in a region of strong interaction.

The results of the experimental measurements of the heat-transfer coefficients at $M = 3{\cdot}5$–$8{\cdot}1$, $T_w = 0{\cdot}62$–$0{\cdot}83$ and $\chi = 4$–24 are given in Chart II (Fig. 4) in the co-ordinate system χ–M^3St_x. These results are within the range $M\sqrt{C}/\sqrt{Re_x} \approx 0{\cdot}3$–$0{\cdot}20$ and were taken for the second and third sections. The value of the recovery factor $r_x = 0{\cdot}95$ obtained experimentally was used to determine the equilibrium temperature, whose value is necessary to calculate the heat-transfer coefficients. The experiments on heat transfer were conducted at

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* The experiments to study the equilibrium temperature were conducted at numbers $1 \lesssim Kn_t \lesssim 3{\cdot}5$.

$1{\cdot}2 \lesssim Kn_t \lesssim 4{\cdot}2$. The same Chart contains the results obtained by the formula derived from the theory of strong interaction [1]:

$$M^3 St_x = St_0 \, \chi^{3/2} \tag{3}$$

where $St_0 = f(\bar{T}_w \gamma)$ is tabulated in [1]. Over the whole range of the strong interaction parameters the experimental data well agree with the theoretical relation (3).

Beside the data on heat transfer described above, data were also obtained at $M\sqrt{C}/\sqrt{Re_x} > 0{\cdot}3$ (mainly, using the first section). These data lie below the curves for strong interaction theory, which fact is explained by the effect of slip. The results of measurment obtained at $M\sqrt{C}/\sqrt{Re_x} > 0{\cdot}3$ are not shown in Chart II (Fig. 4). The analysis which included these data showed that the effect of slip became noticeable only at $M\sqrt{C}/\sqrt{Re_x} > 0{\cdot}3$. This conclusion agrees well with the experimental data [19, 11] obtained in another range of values of the recovery factors and Mach numbers.

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Abstract—The paper contains methods and results of the expcrimental investigations of local heat-transfer coefficients and equilibrium temperatures for a thin flat plate in air flow at zero incidence, with large Mach and low Reynolds numbers.

Résumé—L'article contient les méthodes et les résultats des recherches expérimentales des coefficients locaux de transfert de chaleur et des températures d'équilibre pour une plaque plane mince dans un écoulement d'air à incidence faible, avec de grands nombres de Mach et de faibles nombres de Reynolds.

Zusammenfassung—Diese Arbeit befasst sich mit Methcden und Ergebnissen der experimentellen Untersuchung von lokalen Wärmeübergangszahlen und Gleichgewichtstemperaturen an einer dünnen ebenen Platte (mit Anstellung Null) bei grossen Mach- und niedrigen Reynoldszahlen.